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The analogues of the basic problems of the theory of elasticity for the special 3-d strain in the plates

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1. Introduction

The special case of 3-d strain of a thin symmetric plate was considered in Sharafutdinov (2000) where the author introduced three complex potentials instead of two potentials for the plane case. In the present article two boundary problems for unbounded domains on this plate are stated, they are the analogues of the problems, stated in Muskhelishvili (1966) for the plane domains. The exact solution of the problems for the plate with the symmetric drop-like cut with cusp is obtained. The problems are applicable to the coats of the plane unbounded domains with cuts.

2. Analysis

We consider a thin plate of variable thickness. Let this plate be symmetric with respect to the plane X_1OX_2 and let $\vec{u}=(u_1,u_2,u_3)$ be the vector of displacements at every point of this plate. Let u_1 and u_2 depend only on x_1 and x_2 and let $u_3(x_1,x_2,x_3)=g(x_1,x_2)x_3$. Here (x_1,x_2,x_3) are the coordinates of a point of the plate. This case was investigated in Sharafutdinov (2000), it was proved that $g(x_1,x_2)$ must be harmonic in the corresponding domain. The complex potentials $\phi(z)$, $\psi(z)$ and $f(z)$, $z=x_1+ix_2$, were introduced so that the components of the stress tensor had the following representations:

$$\begin{aligned}
\sigma_{11} &= \frac{1}{2}\mathrm{Re}\, f'(z) - \frac{2\mu}{\lambda+\mu}\mathrm{Re}\,\phi'(z) - \mathrm{Re}\,[\bar{z}\phi''(z)+\psi'(z)],\\
\sigma_{22} &= \frac{1}{2}\mathrm{Re}\, f'(z) - \frac{2\mu}{\lambda+\mu}\mathrm{Re}\,\phi'(z) + \mathrm{Re}\,[\bar{z}\phi''(z)+\psi'(z)],\\
\sigma_{33} &= \frac{2(3\lambda+4\mu)}{\lambda+\mu}\mathrm{Re}\,\phi'(z) - \mathrm{Re}\, f'(z), \quad \sigma_{12} = \mathrm{Im}\,[\bar{z}\phi''(z)+\psi'(z)],\\
\sigma_{13} &= \left[\frac{2(\lambda+2\mu)}{\lambda+\mu}\mathrm{Re}\,\phi''(z) - \frac{1}{2}\mathrm{Re}\, f''(z)\right]x_3, \quad \sigma_{23} = \left[\frac{1}{2}\mathrm{Im}\, f''(z) - \frac{2(\lambda+2\mu)}{\lambda+\mu}\mathrm{Im}\,\phi''(z)\right]x_3.
\end{aligned} \tag{1}$$

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